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Dark energy: a quantum fossil from the inflationary universe?

Joan Solà

High Energy Physics Group, Department ECM, and Institut de Ciencies del Cosmos, Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Catalonia, Spain

E-mail: sola@ifae.es

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Abstract
The discovery of dark energy (DE) as the physical cause for the accelerated expansion of the Universe is the most remarkable experimental finding of modern cosmology. However, it leads to insurmountable theoretical difficulties from the point of view of fundamental physics. Inflation, on the other hand, constitutes another crucial ingredient, which seems necessary to solve other cosmological conundrums and provides the primeval quantum seeds for structure formation. One may wonder if there is any deep relationship between these two paradigms. In this work, we suggest that the existence of the DE in the present Universe could be linked to the quantum field theoretical mechanism that may have triggered primordial inflation in the early Universe. This mechanism, based on quantum conformal symmetry, induces a logarithmic, asymptotically free, running of the gravitational coupling. If this evolution persists in the present Universe, and if matter is conserved, the general covariance of Einstein's equations demands the existence of dynamical DE in the form of a running cosmological term, Λ, whose variation follows a power law of the redshift.

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1. Introduction

Modern cosmology incorporates the notion of dark energy (DE) as an experimental fact that accounts for the physical explanation of the observed accelerated expansion [1, 2]. Although the nature of the DE is not known, one persistent possibility is the 90-years-old cosmological constant (CC) term, Λ, in Einstein's equations. In recent times, one is tempted to supersede this hypothesis with another, radically different, one: namely a slowly evolving scalar field ϕ ('quintessence') whose potential, $V(\phi) \gtrsim 0$, could explain the present value of the DE and whose equation of state (EOS) parameter $\omega_\phi = p_\phi/\rho_\phi \simeq -1 + \dot{\phi}^2/V(\phi)$ is only slightly larger than -1 (hence insuring a negative pressure mimicking the Λ case) [3]. The advantage to think this way is that the DE can then be a dynamical quantity taking different values throughout the history of the Universe. However, this possibility cannot explain why the DE is entirely due to such an ad hoc scalar field and why the contributions to the vacuum energy from the other fields (e.g. the electroweak standard model ones) must not be considered. In short, it does not seem to be such a wonderful idea to invent the field ϕ and simply replace $\rho_\Lambda = \Lambda/8\pi G$ (the energy density associated with Λ, where G is Newton's constant) with $\rho_\phi \simeq V(\phi)$. One has to explain, too, why the various contributions (including the additional one $V(\phi)$!) must conspire to generate the tiny value of the DE density at present—the 'old CC problem' [3]. While we cannot solve this problem at this stage, the dynamical nature of the DE makes allowance for this possibility. Furthermore, since there is no obvious gain in the quintessence idea, we stick to the CC approach, although we extend it to include the possibility of a dynamical ('running') Λ term [4, 5]. The obvious question now is: where this dynamics could come from?

One possibility is that it could originate from the fundamental mechanism of inflation [6], which presumably took place in the very early Universe and could have left some loose end or remnant—kind of 'fossil'—in our late Universe, which we do not know where to fit in now. However, what mechanism of inflation could possibly do that? There is in principle a class of possibilities, in particular see [7, 8], but our source of inspiration here is the quantum theory of the conformal factor, which was extensively developed in [9]. For a recent discussion, see e.g. [10, 11] and references therein. More specifically, we start from the idea of 'tempered anomaly-induced inflation', which was first proposed in [12, 13] (see also [14]). It leads essentially to a modified form of the original Starobinsky model [15]. In the present paper, we push forward the possibility that the mechanism that successively caused, stabilized, slowed down ('tempered') and extinguished the fast period of inflation in our remote past could have left an indelible imprint in the current Universe, namely a very mild (logarithmically) running Newton's coupling G. We show that, if matter is covariantly conserved, this necessarily implies an effective renormalization group (RG) running of the cosmological constant energy density, ρ_Λ, according to a cubic law of $a^{-1} = 1 + z$ during the matter dominated epoch (a being the scale factor and z the cosmological reshift).

2. Anomalous conformal symmetry in cosmology

Following [16], we construct a formulation of the standard model (SM) in curved spacetime which possesses dilatation symmetry, and extend it to local conformal invariance in $d = 4$ [12]. The action of the theory must include conformally invariant kinetic terms and interaction terms. As for scalars φ (e.g. Higgs bosons) we take that their kinetic terms appear in the combination $(1/2)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + (1/2)\xi R\varphi^2$, which is well known to be conformally invariant for $\xi = 1/6$ (after using the non-trivial local conformal transformation law for the scalar of curvature R). The fermion and gauge boson kinetic terms are also well known to be conformally invariant. After the standard set of conformization prescriptions have been applied, the only non-invariant terms are the massive ones. To fully conformize this theory at the classical level, we adhere to the procedure of the Cosmon model [16], where one replaces these parameters by functions of some new auxiliary scalar field χ. This field is a background field, and so within the philosophy of QFT in curved spacetime, it is not submitted (like the metric itself) to quantization. For instance, for the scalar and fermion mass terms in the action we replace

$$\int \mathrm{d}^4x\sqrt{-g}m_\varphi^2\varphi^2 \to \int \mathrm{d}^4x\sqrt{-g}\frac{m_\varphi^2}{\mathcal{M}^2}\varphi^2\chi^2$$
$$\int \mathrm{d}^4x\sqrt{-g}m\bar{\psi}\psi \to \int \mathrm{d}^4x\sqrt{-g}\frac{m}{\mathcal{M}}\bar{\psi}\psi\chi, \tag{1}$$

where $\mathcal{M}$ is an auxiliary mass, e.g. related to a high energy scale of spontaneous symmetry breaking of dilatation symmetry [16]. We expect $\mathcal{M}$ in the range of the grand unified theories (GUT's) or higher: $\mathcal{M} \gtrsim M_X \sim 10^{16}$ GeV, but certainly below the Planck scale $M_P \simeq 1.22 \times 10^{19}$ GeV. Moreover, there is the Einstein–Hilbert (EH) action for gravity itself, $S_{\rm EH}$. With the help of the background field χ, we can conformize it as follows:

$$S_{\rm EH} \to S_{\rm EH}^* = -\frac{M_P^2}{16\pi\mathcal{M}^2}\int \mathrm{d}^4x\sqrt{-g}[R\chi^2 + 6(\partial\chi)^2]. \tag{2}$$

Note that the setting $\chi = \mathcal{M}$ on the conformized action restores the original EH form, as well as all the terms of the original SM action. This setting (kind of conformal unitary gauge [12]) can actually be understood in a more dynamical sense within the context of nonlinearly realized dilatation symmetry [16]. Namely, by reparameterizing $\chi = \mathcal{M}\exp(\Sigma/\mathcal{M})$, the Σ-field just shifts under conformal transformations and behaves as the Goldstone boson (dilaton) of spontaneously broken dilatation symmetry at the high scale $\mathcal{M}$. In this context, the setting $\chi = \mathcal{M}$ can be thought of as χ taking a vacuum expectation value (VEV), with $\Sigma/\mathcal{M} \ll 1$ because Σ performs small oscillations around it. The full conformized classical action of the model becomes invariant under the set of simultaneous transformations

$$(\chi,\varphi) \to (\chi,\varphi)\,\mathrm{e}^{-\alpha}, \qquad g_{\mu\nu} \to g_{\mu\nu}\,\mathrm{e}^{2\alpha}, \qquad \psi \to \psi\,\mathrm{e}^{-3/2\alpha}, \tag{3}$$

for any spacetime function $\alpha = \alpha(x)$ and for all scalar and fermion quantum fields φ and ψ, including the background metric and scalar field χ.

In this context, the generalized form of the vacuum action in renormalizable QFT in curved spacetime is [12]: $S_{\rm vac} = S_{\rm EH}^* + S_{\rm HD}$. Here the first term is the conformal EH term (2), whereas the second contains higher derivatives of the metric and can be expressed in the conformally invariant fashion

$$S_{\rm HD} = \int \mathrm{d}^4x\sqrt{-g}\{a_1C^2 + a_2E + a_3\nabla^2R\}, \tag{4}$$

where, $a_{1,2,3}$ are some parameters, C^2 is the square of the Weyl tensor and E is the Gauss–Bonet topological invariant in $d = 4$. The total action is

$$S_t = S_{\rm matter} + S_{\rm vac} + \bar{\Gamma}, \tag{5}$$

where the part $S_{\rm matter} + S_{\rm vac}$ is classically conformally invariant. However, the one-loop part $\bar{\Gamma}$ is not conformally invariant and constitutes the anomaly-induced action [9–11]. To determine it explicitly, we follow (actually extend) the standard procedure based on reparameterizing the background fields $(g_{\mu\nu}, \chi)$ with the help of the conformal factor σ and a set of (regular) reference fields $(\bar{g}_{\mu\nu}, \bar{\chi})$, as follows:

$$g_{\mu\nu} = \mathrm{e}^{2\sigma}\bar{g}_{\mu\nu}, \qquad \chi = \mathrm{e}^{-\sigma}\bar{\chi}. \tag{6}$$

Through these field redefinitions one can solve the functional differential equation defining the trace anomaly. In the present case, it has an extra term

$$\begin{aligned}\langle T_\mu^\mu\rangle &= -\frac{2}{\sqrt{-g}}g_{\mu\nu}\frac{\delta\bar{\Gamma}}{\delta g_{\mu\nu}} + \frac{1}{\sqrt{-g}}\chi\frac{\delta\bar{\Gamma}}{\delta\chi}\\ &= -\left\{wC^2 + bE + c\nabla^2R + \frac{f}{\mathcal{M}^2}[R\chi^2 + 6(\partial\chi)^2]\right\}.\end{aligned} \tag{7}$$

The one-loop values of the β-functions w, b and c have been established for a long time [17] and depend on the matter content of the model. In particular, $c > 0$ is required for stable inflation [15]. We compute here the one-loop coefficient of the new part, with the following result:

$$f = \frac{1}{3(4\pi)^2}\sum_F N_F m_F^2 + \frac{1}{2(4\pi)^2}\sum_V N_V M_V^2, \tag{8}$$

where m_F and M_V are the various (Dirac) fermion and vector boson masses, respectively, and N_F and N_V are their respective multiplicities (note that scalars do not contribute). We remark that both types of terms in (8) are positive definite, hence we infer the important result that $f > 0$ for all possible quantum matter contributions. Disregarding a conformally invariant term [12], one arrives at the following solution of equation (7) for the anomaly-induced effective action of the combined background fields $g_{\mu\nu}$ and χ:

$$\Gamma_{\text{ind}} = \int d^4x\sqrt{-\bar{g}}\left\{w\bar{C}^2\sigma + b\left(\bar{E} - \frac{2}{3}\bar{\nabla}^2\bar{R}\right)\sigma + 2b\sigma\bar{\Delta}_4\sigma + \frac{f}{\mathcal{M}^2}[\bar{R}\bar{\chi}^2 + 6(\partial\bar{\chi})^2]\sigma\right\} - \frac{3c+2b}{36}\int d^4x\sqrt{-\bar{g}}[\bar{R} - 6(\bar{\nabla}\sigma)^2 - 6(\bar{\nabla}^2\sigma)]^2, \tag{9}$$

where

$$\Delta_4 = \nabla^4 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \tfrac{2}{3}R\nabla^2 + \tfrac{1}{3}(\nabla^\mu R)\nabla_\mu \tag{10}$$

is the fourth-order, self-adjoint, conformal operator acting on scalars.

In the cosmological context, the conformal factor σ is related to the scale factor through $\sigma = \ln a(\eta)$, where $\eta = \int dt/a$ is the conformal time. Furthermore, to better clarify the impact on the EH sector of the total action (5), let us substitute (9) in it and rewrite the final result in the following compact form:

$$S_t = S_{\text{matter}} - \int d^4x\sqrt{-\bar{g}}\frac{M_P^2(1-\tilde{f}\ln a)}{16\pi\mathcal{M}^2}[\bar{R}\bar{\chi}^2 + 6(\partial\bar{\chi})^2] + \text{highderiv.terms}, \tag{11}$$

where we have defined the dimensionless parameter [12]

$$\tilde{f} = \frac{16\pi f}{M_P^2} = \frac{1}{3\pi}\sum_F\frac{N_F m_F^2}{M_P^2} + \frac{1}{2\pi}\sum_V\frac{N_V M_V^2}{M_P^2}. \tag{12}$$

In order to project the standard EH frame (in combination with the higher derivative terms) we set χ to its VEV, $\mathcal{M}$, where conformal symmetry is spontaneously broken; hence, from (6), $\bar{\chi} = \mathcal{M}e^\sigma = \mathcal{M}a$. In conformal time, the flat FLRW metric is conformally flat, so we have $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$ and the terms of S_{HD} trivially decouple from the conformal factor dynamics. The equation of motion for the scale factor can be computed from the functional derivative of (11), $\delta S_t/\delta a(\eta) = 0$, upon reverting to the cosmic time. The exact equation for $a = a(t)$ is a rather complicated, nonlinear, fourth-order differential equation. A numerical, and also an (approximate) analytical, solution is given in [12–14]. The essential analytic result is encapsulated in the 'tempered anomaly-induced solution', which takes the elegant form

$$a(t) = e^{H_P t}\,e^{-\frac{1}{4}H_P^2\tilde{f}t^2}, \tag{13}$$

in which the parameter $\tilde{f}$ is seen to play a fundamental role. Here the scale H_P defines the 'driving force' for the anomaly-induced inflation

$$H_P = \frac{M_P}{\sqrt{-16\pi b}}, \qquad b = -\frac{N_S + 11N_F + 62N_V}{360\cdot(4\pi)^2}, \tag{14}$$

where N_S, N_F, N_V are the number of scalars, Dirac fermions and vector bosons contributing to the one-loop result. We can see that, in this context, primordial inflation is fundamentally associated with the Planck scale and also to the existence of the $b < 0$ coefficient, which emerges as a pure quantum matter effect. In different models of inflation, one finds different energy sources that trigger the inflationary mechanism [6].

Essential in the structure of the solution (13) is the fact that for $\tilde{f} \neq 0$ the inflationary process is progressively slowed down ('tempered' [12]). Thus, one may judiciously suspect that, starting from general stable inflation conditions ($c > 0$), the early Universe should connect gradually with the Friedmann–Lemaître–Robertson–Walker (FLRW) phase. Since $H_P \sim M_P$, we can estimate from (13) that this will occur, roughly, after $4/\tilde{f}$ Planck times ($t_P = 1/M_P \sim 10^{-43}$ s). For a typical matter content of a GUT (say, for $N_F + N_V \sim 100$–1000) and $M_X \gtrsim 10^{16}$ GeV, one can check that $\tilde{f}$ is in the range 10^{-5}–10^{-3} and, hence, the inflationary period should typically stop at around a hundred thousand Planck times, at most, i.e. at $t \sim 10^{-38}$ s. This dating of the inflationary epoch lies in the expected range of most inflationary models [6].

Equation (11) suggests that the Newton coupling $G \sim 1\big/M_P^2$ evolves with the scale factor since $M_P^2 \to M_P^2(1 - \tilde{f}\ln a)$. Defining $\tau \equiv -\ln a$, the dimensionless parameter (12) can be interpreted as the coefficient of the β-function driving the renormalization group equation (RGE) for the effective ('running') Newton's coupling

$$\frac{\partial}{\partial\tau}\frac{1}{\bar{G}} = \beta_{G^{-1}}(\bar{G}), \qquad \bar{G}(\tau = 0) = G_0, \tag{15}$$

where $\bar{G} = \bar{G}(G, \tau)$ is a function of the renormalized coupling G at the scale τ, and $G_0 \equiv 1\big/M_P^2$ is the current value. At the one-loop level,

$$\beta_{G^{-1}}^{(1)} = \frac{\tilde{f}}{G_0} = \frac{1}{3\pi}\sum_F N_F m_F^2 + \frac{1}{2\pi}\sum_V N_V M_V^2. \tag{16}$$

Being $\tilde{f} > 0$, it follows that the parameter $\bar{G}^{-1}$ is infrared free and hence the inverse one, $\bar{G}$ (the running Newton's coupling), is asymptotically free. This can be seen from the explicit solution of (15) at one-loop level

$$\bar{G}(a) = \frac{G_0}{1 - \tilde{f}\ln a} = \frac{G_0}{1 + \tilde{f}\ln\mu}, \tag{17}$$

where we observe that $\bar{G}(\mu) \to 0$ for $\mu \equiv 1/a \to \infty$.

3. Running of G and Λ in the present universe

The logarithmic running of the gravitational coupling (17) is controlled by the parameter $\tilde{f}$. Such slow evolution may appear nowadays as a kind of 'fossil inertia', reminiscent of the early inflationary times. Let us note, however, that the potential infrared effects on the value of $\tilde{f}$ could not be taken into account in the above calculation. Therefore, we do not know the precise prediction for $\tilde{f}$ at the present time and, in this sense, it can be treated as a phenomenological parameter. Requiring that it should not alter significantly the standard picture, we may arguably suspect that it is still a small number. We will assume that at low energies (i.e. in the present Universe) it satisfies $0 < \tilde{f} \ll 1$. Due to the logarithmic character of the law (17), the running of the gravitational coupling should be very smooth and virtually undetectable. We remark that its variation should be understood at a global cosmological level, not as a local one.

The conformal anomaly, being a short distance effect associated with inflation in the early Universe, should not distort the formal structure of Einstein's equations at very large distances.

Thus, we expect essentially the same low energy gravitational theory in the present Universe. As already mentioned in the previous section, the full equation of motion is a fourth-order, nonlinear, differential equation. When the inflationary phase has stopped, we must recover the FLRW Universe in the radiation epoch, and therefore the scale factor grows approximately as $t^{1/2}$. One finds that all higher order terms in the aforementioned equation of motion decay as $1/t^4$ whereas the standard ones decay as $1/t^2$ [12–14]. As a result the effect of the higher order terms in the present Universe is negligible. Moreover, the terms which are proportional to $\tilde{f}$ can all be absorbed in M_P according to $M_P^2 \to M_P^2(1-\tilde{f}\ln a)$ [12]. Therefore, only a small renormalization of the parameter G remains in the infrared epoch as a function of the scale factor. Does this mean that we cannot get any hint of the primordial dynamics of the early Universe? Not necessarily so. Let us consider the possible impact on the cosmological term.

From the above considerations, we may assume that at the present time the gravitational field equations are Einstein's equations with a non-vanishing Λ term and a slowly running Newton's coupling G. Let us first confirm that the Λ term must indeed be present in this framework as a consistency requirement. Modeling the isotropic Universe as a perfect fluid, we have Einstein's equations in the form

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = 8\pi\bar{G}\tilde{T}_{\mu\nu} \equiv 8\pi\bar{G}[(\rho_\Lambda - p_m)g_{\mu\nu} + (\rho_m + p_m)U_\mu U_\nu], \tag{18}$$

where ρ_m and p_m are the matter density and pressure. Consider now the Bianchi identity satisfied by the Einstein's tensor on the lhs of equation (18). It leads to the following generalized, covariant, local conservation law: $\nabla^\mu(\bar{G}\tilde{T}_{\mu\nu}) = 0$, where we recall that $\bar{G}$ is not constant in this framework. We can readily evaluate this law in the FLRW metric. If we project the $\nu = 0$ component of it, we find

$$\frac{\mathrm{d}}{\mathrm{d}t}[\bar{G}\,(\rho_m + \rho_\Lambda)] + \bar{G}H\alpha_m\rho_m = 0, \qquad \alpha_m \equiv 3(1+\omega_m), \tag{19}$$

where we have introduced the EOS of matter $p_m = \omega_m\rho_m$, with $\omega_m = 0, 1/3(\alpha_m = 3, 4)$ for cold matter and relativistic matter (radiation), respectively. In the following we adhere to the canonical assumption that matter is conserved, namely

$$\frac{\mathrm{d}\rho_m}{\mathrm{d}t} + \alpha_m\rho_m H = 0 \to \rho_m(a) = \rho_m^0 a^{-\alpha_m}, \tag{20}$$

where $\rho_m^0 = \rho_m(a=1)$ is the matter density at the present time. Substituting (20) into the generalized conservation law (19), we find

$$(\rho_m + \rho_\Lambda)\,\mathrm{d}\bar{G} + \bar{G}\,\mathrm{d}\rho_\Lambda = 0. \tag{21}$$

Admitting that $\bar{G}$ is variable as in (17), this differential relation implies $\rho_\Lambda \neq 0$. Moreover, it cannot be satisfied by a strictly constant $\rho_\Lambda \neq 0$, unless $\rho_m(a) = -\rho_\Lambda$ at all times, which would of course entail a static Universe! Therefore, we must have $\rho_\Lambda = \rho_\Lambda(a)$ as well! In other words, the variable $\bar{G}$ induces a non-vanishing ρ_Λ in our Universe, and the latter must necessarily be dynamical. To determine $\rho_\Lambda = \rho_\Lambda(a)$, let us substitute (17) and (20) into (21) and rearrange terms. The final result is

$$\frac{\mathrm{d}\rho_\Lambda}{\mathrm{d}a} + P(a)\rho_\Lambda = Q(a), \tag{22}$$

where the functions P and Q read

$$P(a) = \frac{\tilde{f}}{a(1-\tilde{f}\ln a)}, \qquad Q(a) = -\frac{\tilde{f}\rho_m^0}{a^{\alpha_m+1}(1-\tilde{f}\ln a)}. \tag{23}$$

The exact solution can be obtained by quadrature as follows:

$$\rho_\Lambda(a) = (1 - \tilde{f}\ln a)\rho_\Lambda^0 - \tilde{f}\rho_m^0(1 - \tilde{f}\ln a)\int_1^a \frac{\mathrm{d}x}{x^{\alpha_m+1}(1 - \tilde{f}\ln x)^2}, \tag{24}$$

where $\rho_\Lambda^0 = \rho_\Lambda(a = 1)$ is the value of the CC density at present. The last integral cannot be performed in terms of elementary functions. However, since we expect $\tilde{f} \ll 1$, we can just present the result at leading order in $\tilde{f}$ as follows (if we also neglect $\tilde{f}\ln a \ll 1$):

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\tilde{f}\rho_m^0}{\alpha_m}[(1+z)^{\alpha_m} - 1], \tag{25}$$

where for convenience we have re-expressed the result in terms of the cosmological redshift, $z = \mu - 1 = (1 - a)/a$. From (25) we see that, in the matter dominated epoch ($\alpha_m = 3$), the cosmological term evolves as an approximate cubic power law of the redshift. This result is remarkable and encouraging; it tells us that, despite the extremely slow logarithmic running of the gravitational coupling with the redshift, the dark energy (in this case, a running cosmological term) evolves like an (approximate) power law of the redshift. The cosmological term, therefore, finally reveals as the truly detectable 'fossil' (in this case, a 'fossil energy') that emerges from this kind of inflationary-inspired scenario. Even if $\tilde{f}$ is as small at present as indicated by the high energy computation ($\tilde{f} \sim 10^{-5} - 10^{-3}$), there is a good chance for testing this model by considering the sensitivity of the cosmological perturbations to a running ρ_Λ, e.g. following the approach of [18]. If, however, $\tilde{f} \sim 10^{-3}$–10^{-2}, the running of ρ_Λ could already be detected from a dedicated EOS analysis of the DE, see [23].

It is useful to write the corresponding generalized Friedmann's equation (with vanishing spatial curvature) in this model. The result is the following:

$$H^2(a) = \frac{8\pi}{3}\frac{G_0}{1 - \tilde{f}\ln a}\left[\frac{\rho_m^0}{a^{\alpha_m}} + \rho_\Lambda(a)\right], \tag{26}$$

where $\rho_\Lambda(a)$ is given by (24). To fully analyze the cosmological consequences of this model, one has to cope with this complete formula [24]. However, to order $\tilde{f}$, and considering cosmological epochs not very far in the future (therefore, neglecting again $\tilde{f}\ln a \ll 1$), it boils down to

$$H^2(z) = \frac{8\pi G_0}{3}\left[\rho_m^0(1+z)^{\alpha_m} + \rho_\Lambda(z)\right], \tag{27}$$

with $\rho_\Lambda(z)$ given by (25). Using equation (27) and working within the same approximation, we may rewrite (25) as

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\tilde{f}}{\alpha_m}\frac{3M_P^2}{8\pi}\left[H^2(z) - H_0^2\right], \tag{28}$$

where $H_0 = H(z = 0)$. This equation is formally identical to the one obtained in [19–21] where the running scale $\mu = H$ was assumed, instead of $\mu = 1/a$. The parameter ν introduced in these references can be identified here with $\tilde{f}/\alpha_m$. This correspondence allows us to immediately transfer the primordial nucleosynthesis bounds on the parameter ν, obtained in [21] for a G-running model similar to the present one, to the parameter $\tilde{f}$. This result confirms that $\tilde{f}$ cannot be larger than 10^{-2}. In the following section, we further explore the interesting connections with previous frameworks.

4. Tracking the running of the cosmological parameters physically

Although we have found that the scale factor (or equivalently, $\mu = 1/a$) is the original running scale appearing in our framework, it is useful to investigate if there are other relevant running scales, with more physical meaning than a, that could be useful to track the evolution of the cosmological parameters. In the previous section, we have already mentioned the energy scale defined by the Hubble function, $\mu = H$. This scale was originally proposed in [4] and further exploited in [19–21], see also [22]. Consider now the periods of the cosmic history more accessible to our observations, i.e. the matter and radiation dominated epochs. In this case, the approximate formula (25) applies to within very good accuracy. Then, from (27), we have

$$\ln \frac{H^2(z)}{H_0^2} \simeq -\alpha_m \ln a, \tag{29}$$

where we assume that z is sufficiently high such that the CC is subdominant (recall that $\tilde{f} \ll 1$). The meaning of equation (29) is that, in virtually all our observable past, the running of G in terms of the scale factor can be traced by a useful physical observable: the expansion rate H. In this way, the effective coupling $\bar{G}$ in (17) can approximately be rewritten as a running function of H:

$$G(H/H_0) = \frac{G_0}{1 + (\tilde{f}/\alpha_m) \ln \left(H^2/H_0^2\right)}. \tag{30}$$

This result is encouraging because it nicely fits with the previous result obtained in the alternative framework of [21], that is, provided we use (again) the correspondence $\tilde{f} \leftrightarrow \alpha_m \nu$ between the basic parameters of the two frameworks. At the same time, we have the running of the CC term as a function of the expansion rate

$$\rho_\Lambda(H/H_0) = \rho_\Lambda^0 + \frac{\tilde{f} \rho_m^0}{3} \left(\frac{H^2(z)}{H_0^2} - 1 \right). \tag{31}$$

As it is patent from this equation, the running of the CC term can be traced by H during the entire matter and radiation dominated epoch up to the present day. This includes, in particular, the full range of the supernovae observations.

From the foregoing, we see that $\mu = H$ acts as an alternative running scale that tracks the evolution of the cosmological parameters G and ρ_Λ. Although the primary evolution of these parameters is formulated in terms of the scale factor, the latter is not physically measurable. In contrast, the expansion rate H is a physical observable, which we are measuring nowadays with an increasing level of precision. In this sense, the evolution of the cosmological parameters can be better tracked through the evolution of H, whenever possible. Furthermore, the use of H as a running scale allows the present cosmological model to naturally connect with the previous RG formulations [19–21, 23] and, at the same time, to benefit from the various phenomenological opportunities described there to identify the dark energy as a dynamical cosmological term. A possible advantage of the present approach is that it preserves essentially all the nice features of the previous ones and suggests a potential connection with the primordial physics of the early Universe. It is in this sense that the DE that we have detected in our old Universe could be viewed as a 'fossil' of the very early times; in fact a 'quantum fossil' because the non-zero value of the coefficient $\tilde{f}$ is related to the quantum effects of matter particles, see equation (12).

Let us note that the trading of a for H ceases to hold when the Universe becomes highly dominated by the CC since, then, the Universe is essentially in the de Sitter phase, which means that H becomes constant even though the scale factor starts to grow exponentially. Clearly, in such circumstances H is not a good tracer of a. Therefore, in the future, when the control of the evolution is overtaken by an approximately constant cosmological term Λ_0, the expansion rate takes the value $H \simeq \sqrt{\Lambda_0/3} = H_0\sqrt{\Omega_\Lambda^0} \equiv H_*$. While this regime persists, we have $\ln a = H_* t$ and the original running law (17) cannot be mimicked as in (30), but as follows:

$$G(t) \simeq \frac{G_0}{1 - \tilde{f} H_* t}. \tag{32}$$

During the quasi-de Sitter regime, the physical scale parameter tracking the running of G is the cosmic time; equivalently, the energy scale is $\mu = 1/t$. In this case, H and ρ_Λ remain essentially constant whereas G increases with t as indicated above. However, this situation will not last forever; one can show from the full structure of the expansion rate (26)—with $\rho_\Lambda(a)$ given by (24)—and from the equation for $\ddot{a}$ (the acceleration), that there is a remote future instant of the cosmic where the Universe will arrive at a turning point in its evolution. We shall not dwell here on the details of that remote future epoch [24]. It suffices to say that the Universe somehow will recreate in the distant future the tempered inflation process that it underwent in the past, in the sense that the present and future state of slow inflation will also cease, roughly after $1/\tilde{f} \sim 10^4$ Hubble times, namely when the turning point will be approached.

5. Soft decoupling and running of ρ_Λ in the present universe

It is important to emphasize that, in this framework, the running of the cosmological term is actually tied to the running of the gravitational coupling. This can be better seen if we rewrite the Bianchi identity (21) as follows:

$$\frac{\mathrm{d}\rho_\Lambda}{\mathrm{d}\tau} = G(\rho_m + \rho_\Lambda)\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{1}{G}\right) = \frac{3}{8\pi}H^2\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{1}{G}\right), \tag{33}$$

where in the second equality we have used Friedmann's equation in the flat case (G being here, of course, $\bar{G} = \bar{G}(a)$). As we have discussed in section 4, use of the expansion rate as the running scale is adequate for most practical purposes at present and, in addition, it enables us to track the running of the parameters in terms of a direct physical observable. Therefore, let us further transform the RGE (33) in terms of the more physical running scale H through the relation (29). The latter leads to $\mathrm{d}\tau = -\mathrm{d}\ln a = (2/\alpha_m)\mathrm{d}\ln H$. Using the one-loop result for the RGE of G^{-1}, equation (15)–(16), we may express the desired RGE as follows:

$$\frac{\mathrm{d}\rho_\Lambda}{\mathrm{d}\ln H} = \frac{3\nu}{4\pi}H^2 M_P^2, \qquad \nu \equiv \frac{\tilde{f}}{\alpha_m}. \tag{34}$$

The obtained equation (34) has exactly the required form that we suggested on different grounds in previous approaches to the RG evolution of the cosmological term (see e.g. [19–21]). As a result, we obtain a possible unified description of the early and late history of the Universe in RG terms. Moreover, the physical interpretation of ν in the present framework is physically the same as in the previous approach, except that here we have found a possible connection of this parameter with the mechanism of primeval inflation. Indeed, with the help of (12), we can rewrite ν in (34) as follows:

$$\nu = \frac{1}{12\pi}\frac{M^2}{M_P^2} \tag{35}$$

with

$$M^2 = \frac{4}{\alpha_m}\sum_F N_F m_F^2 + \frac{6}{\alpha_m}\sum_V N_V M_V^2 \equiv \sum_i c_i M_i^2. \tag{36}$$

Here M_i are the masses of all the matter particles contributing in the loops. Equations (35) and (36) adopt the general form that we postulated for the RGE of the cosmological term in

[4, 19–21]. We see, remarkably enough, that the heaviest particles provide the leading contribution to the running of ρ_Λ. This feature is what we called 'soft-decoupling' in these references, in the sense that the cosmological term evolution satisfies, in contrast to the other parameters in QFT, a renormalization group equation that is driven in part by the heaviest masses M_i and in part by the physical running scale $\mu = H$. The running law for the Λ term, thus, follows a kind of generalization of the decoupling theorem [26]. This is related to the fact that ρ_Λ is a dimension-four parameter. Since there appears no contribution on the rhs of (34) that is entirely driven by the masses, otherwise it should be of the type $\sim M_i^4$—and hence disastrous from the phenomenological point of view [4]—the leading effects are of the mixed form $\sim H^2 M_i^2$. For fields whose masses are of order of the Planck mass ($M_i \lesssim M_P$) the contribution at the present time is of the order $H_0^2 M_P^2$. Recalling that $H_0 \sim 10^{-42}$ GeV, we find that the value of $H_0^2 M_P^2$ falls just in the ballpark of the current value of the CC density, $\rho_\Lambda^0 \sim 10^{-47}\,\text{GeV}^4$. The running of ρ_Λ through (34) is, therefore, smooth and of the correct order of magnitude. To be precise, that RGE tells us that the typical variation of ρ_Λ as a function of H is, at any given time in the cosmic history, of the order of ρ_Λ itself. Let us note that the masses M_i could be substantially smaller than $M_P \sim 10^{19}$ GeV and still get a sizeable effect in the running of the CC. For example, assume that there is physics just at a GUT scale M_X a few orders of magnitude below the Planck scale. This would indeed be the case if we assume that there is a large multiplicity in the number of particles involved in that GUT scale. For example, take $\tilde{f} \sim \nu \sim 10^{-4}$, then from (35)–(36) we have

$$\sum_i c_i M_i^2 = 12\pi M_P^2 \nu \sim 10^{36}\,\text{GeV}^2. \tag{37}$$

If we assume that the number of heavy degrees of freedom, $M_i \sim M_X$, is of order of a few hundred, it follows that $M_X \sim 10^{16}$ GeV. In other words, in this case one could entertain the possibility that the origin of the RG cosmology could bare some relation to the physics near the typical SUSY–GUT scale.

To summarize this section, we have found that the soft-decoupling terms $\sim H^2 M_i^2$ are the leading ones determining the running of ρ_Λ. We obtain no $\sim M_i^4$ contributions at all. The solution of (34) that satisfies the boundary condition $\rho_\Lambda(H = H_0) = \rho_\Lambda^0$ is just equation (28), as expected. We emphasize that the previous interpretation is based on assuming that the computation of $\tilde{f}$ in section 2 can be applied to the present time. As we already warned, there might be infrared effects that could distort this picture, but we have assumed that $\tilde{f}$ will remain small and may not be essentially different from what we have found. In particular, within a more physical RG scheme, and on the grounds of the decoupling theorem [26], we expect ordinary decoupling corrections on the rhs of (15) and (34), namely corrections of the form $(\mu/M_i)^n$. However, if the scale $\mu = H$ (defining the typical cosmic energy of the FLRW models) could be used for a physical description of the running of the cosmological parameters, these corrections should be negligible [24].

6. Conclusions

In this work we have suggested that the presence of dynamical dark energy (DE) in the current Universe is actually a consistency demand of Einstein equations under the two assumptions of: (i) matter conservation and (ii) the existence of a period of primordial inflation in the early Universe, especially when realized as 'tempered anomaly-induced inflation'. Based essentially on the previous works [12–14] and on the general setting of the quantum theory of the conformal factor [9, 10], we have found that if the inflationary mechanism is caused by quantum effects on the effective action of conformal quantum field theory in curved spacetime, then the gravitational coupling G becomes a running quantity of the scale factor, $G(a) = G_0/(1 - \tilde{f} \ln a)$, $\tilde{f}$ being the coefficient of the β-function for the conformal Newton's coupling. The effect of this coupling on the inflationary dynamics is to efficiently 'temper' the regime of stable inflation presumably into the FLRW regime. The rigorous high energy calculation of $\tilde{f}$ in QFT in curved spacetime shows that both fermions and bosons produce non-negative contributions ($\tilde{f} \geqslant 0$). As a consequence, G becomes an asymptotically-free coupling of the scale factor. Intriguingly enough, we have suggested the possibility that this running might persist in the present Universe and, if so, it could provide a *raison d'être* for the existence of the (dynamical) DE, which would appear in the form of running cosmological vacuum energy ρ_Λ. In fact, the logarithmic evolution of G induces a power-law running of ρ_Λ, which is essentially driven by the soft-decoupling terms $\sim H^2 M_i^2$ (hence by the heaviest particle masses). The result is a Universe effectively filled with a mildly-dynamical DE, which can be perfectly consistent with the present observations.

To summarize, from the point of view of the 'RG-cosmology' under consideration, the current Universe appears as FLRW like while still carrying some slight imprints of important physical processes that determined the early stages of the cosmic evolution. Most conspicuously, the smooth dynamics of G and ρ_Λ can be thought of as 'living fossils' left out of the quantum field theoretical mechanism that triggered primordial inflation. Remarkably, this framework fits with previous attempts to describe the renormalization group running of the cosmological term [4, 19–23, 25, 27] and could provide an attractive link between all stages of the cosmic evolution. It is reassuring to find that there is a large class of RG models behaving effectively the same way. Differences between them could probably be resolved at the level of finer tests, such as those based on cosmological perturbations and structure formation. For example, in [18, 28] it is shown that the study of cosmological perturbations within models of running cosmological constant puts a limit on the amount of running, which is more or less stringent depending on the peculiarities of the model. Similarly, a particular study of perturbations would be required in the present framework (which includes the variation of both Λ and G) to assess the implications on the parameter $\tilde{f}$. This study is beyond the scope of the present work.

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